

**NAME:**

## Math 125 Exam 3

**Instructions:** WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. Note that you must do at least 10 problems correctly to get 100. Write neatly and legibly in the space provided. **SHOW YOUR WORK!**

### Core Problems

1. Convert to radians if the angle  $\theta$  is given in degrees, otherwise convert to degrees.

a)  $\theta = 20^\circ$  [2 pts]

b)  $\theta = -3^\circ$  [2 pts]

c)  $\theta = -\frac{\pi}{9}$  [2 pts]

d)  $\theta = 2$  (radians) [2 pts]

e)  $\theta = \pi^\circ$  (degrees) [2 pts]

2. Solve the equation  $e^{3x} - 3e^{2x} = 10e^x$  for  $x$ . [10 pts]

3. a) Use the properties of logarithms to expand the expression  $\log \frac{100x^3\sqrt[3]{5-x}}{2(x+6)^3}$   
[5 pts]

- b) Use the properties of logarithms to condense the expression  
 $3\ln(\sqrt[3]{x-5}) + 4\ln(\sqrt[4]{x+5}) - \ln(x^2 - 25)$  [5 pts]

4. If  $(-2, \sqrt{5})$  is a point on the terminal side of an angle  $\theta$ , find the exact value of the following trigonometric functions.

- a)  $\sin \theta$  [2 pts]

b)  $\cos \theta$  [2 pts]

c)  $\tan \theta$  [2 pts]

d)  $\csc \theta$  [2 pts]

e)  $\sec \theta$  [2 pts]

5. Sighting the top of a building, a surveyor measured the angle of elevation to be  $45^\circ$ . The transit is 6 feet above the ground and 200 feet from the building. Find the building's height. [10 pts]

6. Use your knowledge of the graph of the function  $f(x) = \log_2 x$  to sketch the graph of the function  $g(x) = -1 + \log_2(x - 2)$ . What is the graph's x-intercept? What is the vertical asymptote? Determine domain and range. [10 pts]

7. Solve the logarithmic equation  $\log_3(x-4) + \log_3(x+4) = 2$ . Be sure to reject any value of  $x$  that is not in the domain of the original logarithmic expressions. [10 pts]

8. Find the exact value of  $\sin\left(\frac{\sqrt{2}\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)$ . Note that you must give the exact answer to get full credit. [10 pts]

9. Find the reference angle for each angle.

a)  $210^\circ$  [2 pts]

b)  $\frac{11\pi}{4}$  [2 pts]

c)  $-250^\circ$  [2 pts]

d)  $\frac{-13\pi}{3}$  [2 pts]

e)  $351^\circ$  [2 pts]

10. A bicycle has wheels, whose diameter is 1 meter in length. If the wheels rotate at a rate of 1300 revolutions per hour, how far will the bicycle have traveled in 30 minutes? [10 pts]

**Extra-Credit**

11. a) Prove the identity  $\log_a MN = \log_a M + \log_a N$  [2 pts]

b) Prove the identity  $\log_a \frac{M}{N} = \log_a M - \log_a N$  [2 pts]

c) Prove the identity  $\log_a M^r = r \log_a M$  [2 pts]

d) Prove the identity  $\log_a M = \frac{\log_b M}{\log_b a}$  [4 pts]